5.1 HW 6 (n! on 151), 8, 14, 18, 28, 38, 40

6) Prove that 1 · 1! + 2 · 2!+· · ·+*n* · *n*! = *(n* + 1*)*! – 1 whenever *n* is a positive integer.

**Basis Step: n x n! = (n+1)! – 1**

**P(1): 1x1 = 1+1 – 1 = 1, so 1 = 1 for P(1).**

**Because P(1) is true, this shows the basis step.**

**Inductive Step: States P(k), where P(k) is 1 · 1! + 2 · 2!+· · ·+*n* · *n*! = *(n* + 1*)*! – 1 is true.**

**We must show that if P(k) is true, then P(k+1) is also true.**

**To do this, add (k +1) x (k + 1)! to each side.**

**(K x k)! + (k+1)x(k+1)! = (k+1)! -1 + (k+1) x (k+1)!**

**= (k+1)![1 + k + 1] – 1**

**= (k+1)!(k + 2) -1**

**= (k+2)! - 1**

**This shows the inductive hypothesis is true and completes the inductive argument. By showing the basis and inductive argument to be true, we can conclude that P(n) is also true.**

8) Prove that 2 − 2 · 7 + 2 · 72 −· · ·+2*(*−7*)n* = *(*1 − *(*−7*)n*+1*) /* 4 whenever *n* is a nonnegative integer.

**Basis Step: 2*(*−7*)n* = *(*1 − *(*−7*)n*+1*) /* 4**

**P(0) gives us 2 = 2, proving that the basis step is true**

**Inductive Step: We must show that if P(k) is true, then P(k+1) is also true.**

**Add 2*(*−7*)n+1* to both sides:**

**2*(*−7*)n* + 2*(*−7*)n+1 =((*1 − *(*−7*)n*+1*) /* 4) + 2*(*−7*)n+1***

**= 1 − *(*−7*)n*+1 *+ 8(-7)n+1/ 4***

**= 1 + 7(-7)n+1/4**

**= 1 + (-7)n+2/4**

**This shows the inductive hypothesis to be true where if P(k), then P(k+1). Because we have shown the basis and inductive steps to be true, we can conclude that the statement is true.**

14) Prove that for every positive integer *n*, = *(n* − 1*)*2*n*+1 + 2.

**Basis Step:**

**P(1): 2 = 2**

**Because 2 = 2, we have shown that the basis step, P(1) is true**

**Inductive Step:**

**Show that if P(k) is true, then P(k+1) is also true**

**Add (k+1)2n+1 to each side, making the right side:**

**(k-1)2k-1 + 2 + (k+1)2k+1 = 2k+1(k-1+k+1) + 2**

**=2k+1(2k) + 2**

**=2k+2(k) + 2**

**This shows that the inductive hypothesis is true for P(k), and that P(k+1) must also be true. Because we have shown the basis and inductive steps to be true, we can conclude that the statement is true.**

18) Let *P(n)* be the statement that *n*! *< nn*, where *n* is an integer greater than 1.

**a)** What is the statement *P(*2*)*?

**2 < 4 or just 2! < 22**

**b)** Show that *P(*2*)* is true, completing the basis step of the proof.

**Because (1 x 2) < (2)(2) , we have shown the basis step that P(2) is true.**

**c)** What is the inductive hypothesis?

**For all positive integers n, where n is greater than 1, and P(n) is true, then P(n+1) is true. k+1! < k + 1k+1**

**d)** What do you need to prove in the inductive step?

**I need to prove that for every n, if P(n) is true, then P(n+1) is also true.**

**e)** Complete the inductive step.

**K!(k + 1) < k2(k+1) < (k + 1)k (k+1) = (k+1)k+1**

**f )** Explain why these steps show that this inequality is true whenever *n* is an integer greater than

**These steps show both the basis step and inductive step to be true, showing that the statement is true.**

28) Prove that *n*2 − 7*n* + 12 is nonnegative whenever *n* is an integer with *n* ≥ 3.

**Basis Case:**

**P(3): 9 – 21 + 12 = 0, which is a nonnegative integer.**

**Because P(3) is true, we can say that the base case is true.**

**Inductive Case:**

**If P(k) is true, show that P(k+1) is also true.**

**Insert (k+1) to the equation**

**(k+ 1)2 – 7(k + 1)+12 = k2 + 2k +1 – 7k + -7 + 12**

**= (k2 -7k + 12) + (2k – 6)**

**= (k2 -7k + 12) + 2(k – 3)**

**2(k-3) will be non-neg whenever k > or = to 3, and (k2 -7k + 12) has already been proven for P(k).**

**We have shown that the inductive hypothesis, if true for P(k), is also true for P(k+1). By proving the basis case and the inductive case true, we can conclude that the statement is true.**

38) Prove that if *A*1*,A*2*, . . . , An* and *B*1*,B*2*, . . . , Bn* are sets such that *Aj* ⊆ *Bj* for *j* = 1*,* 2*, . . . , n*, then Un j = 1 Aj ⊆ Un j=1 Bj

**Basis Step: P(1) shows A1 ⊆ B1**

**Inductive Step: P(k) is T where A1 ⊆ B1, then Uk+1, j = 1, Aj ⊆ U k+1 j =1, Bj.**

**X is an arbitry are element of (Uk+1, j=1, Aj) U Ak + 1, where x will be an element of the first or second.**

**If it is of the first, then we can clearly conclude x is an element of U k+1 j =1, Bj. If it is of the second, we can say that x is an element of Bk + 1 because Ak+1 ⊆ Bk+1.**

**We have thus shown the inductive hypothesis P(k) is true, and that then P(k+1) must also be true. Because we have shown the basis step and the inductive step to be true, we can conclude that the original statement is true.**

40) Prove if A1, A2, … An, and B1, B2, … Bn are sets such that Aj ⊆ Bj for j = 1,2, …n then (A1 ∩ A2 ∩ … ∩ An) U B = (A1 U B) ∩ (A2 U B) ∩ … ∩ (An U B)

**Basis Step: P(1): A1 U B = A1 U B is true, and proves the Basis step.**

**Inductive Step: If we assume P(k) is true, then we must show that P(k+1) is also true.**

**This gives us (A1 ∩ A2 ∩ … ∩ Ak ∩ Ak+1) U B**

**= [(A1 ∩ A2 ∩ … ∩ Ak) ∩ Ak+1] U B - Associative Law**

**= [(A1 ∩ A2 ∩ … ∩ Ak) U B] ∩ (Ak+1 U B) - Distributive Law**

**= (A1 U B) ∩ (A2 U B) ∩ … ∩ (Ak U B) ∩ (Ak+1 U B)**

**We have thus shown that the inductive hypo P(k) is true, and that P(K+1) then must also be true. We have then shown both the basis case and the inductive case, showing that the original statement is true.**